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ON THE RELATION OF THE DOPPLER EFFECT TO KIRCHHOFF'S LAW

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With such an abundance of literature concerning the so-called "Doppler effect" and Kirchhoff's law in spectral analysis, attention has been paid mainly, and I should say only, to the displacement of the lines, resulting from any *change in the optical path connecting the source of light or the absorbing medium with the observer.* Even in books so complete and of such great authority as the "Traité d'Optique" by Mascart, one reads in Volume III, page 92, "Le résultat ne dépend que de le vitesse relative de l'astre et de l'observateur," a statement which is indeed true if applied to the position of emission lines of the spectrum, but which *does not contain the whole truth,* inasmuch as emission lines can be modified by the relative velocity between the source of light and the absorbing medium independently of the velocity with respect to the observer.

I arrived at this conclusion by considering the relation between Kirchhoff's law and the Doppler-Fizeau effect. The former has been announced as absolutely establishing that a cooler gas absorbs the vibrations of light having the same period or the same wavelength as those emitted by itself when radiating; and H. Fizeau (*Annales de Chimie et de Physique,* **19,** 211, 1870) gave scientific accuracy to Doppler's statement, showing that the effect of a high velocity between the source of light and the observer, would be not the change in color but the displacement of the spectral lines, due to the apparent change in the period with which they reach the observer. Hence just as the refraction and interferences are *functions of the relative period* with which the vibrations arrive at the instrument, *so also will the absorption correspond to the relative period with which the vibrations reach the absorbing gas.*

As a direct consequence of this relation, we could have a layer of cooler sodium vapor, which if moving fast enough, would not absorb at all the two yellow lines of the sodium flame. In general if a gas A, emitting vibrations of absolute period p_0 moves with a velocity v with respect to the source of light, then the original absolute period of the vibrations absorbed by it, will be given by the equation $p_x = p_o \left(1 \mp \frac{x}{c}\right)$, c being the velocity of light and x being determined by the condition equation $p_o \left(1 \mp \frac{x}{c}\right) \left(1 \pm \frac{v}{c}\right) = p_o$, from which, neglecting the term of second order $-\frac{xv}{c^2}$, we get $x = v$ and $p_x = p_o \left(1 \mp \frac{v}{c}\right)$. I have found nothing about the subject in current text-books on optics, such as Drude, Preston, Wood, Schuster, etc., or in those dealing more specifically with spectrum analysis applied to astronomy, as for instance, "Spectroscopie Astronomique," by P. Salet, Paris 1909. It was only in a remarkable monograph, "Das Dopplersche Princip," by Dr. H. Konen in Bonn and published as Chapter VII in the second volume of H. Kayser's "Handbuch der Spectroscopie" that I discovered a hint of my own conclusion.

Curiously enough, Van der Willigen, probably the first to whom the idea occurred that a cooler gas if in motion would not absorb the same vibrations as when it is at rest, held this conclusion as "étrange" and used it as an irrefutable argument against Doppler's principle, as shown in his work, "Sur la fausseté de la proposition que les refractions des rayons lumineux est modifiée par le movement de la source lumineuse et du prisme," to which Konen refers.

One could have hoped that Konen would discuss fully the matter in his thesis, but the way he answers Van der Willigen's argument, that if the above conclusion were true, we could not have the reversal of the bright lines in the solar spectrum, shows that even Konen himself did not pay much attention to the subject. He simply answers that Van der Willigen did not consider that in the case of the Sun, the photosphere and the reversing layer are relatively at rest. This does not seem to be the proper answer, however, because even if the absorbing layer were moving as a whole, the absorption lines would appear the same, with their *relative positions* unchanged, altho the absorbed vibrations of the continuous spectrum would be different. A little further on and in a half-page paragraph (the monograph has about one hundred pages) Konen touches again on the matter and points out the consequence that sodium vapor, if moving fast enough, would not absorb the bright lines characteristic of this element. He adds also that "möglicher Weise, würden sich derartige Vorgange unter Umständen in Sonnenspectrum zeigen" without coming to any application.

Altho when I found this statement, the analysis I submit below was already written, I am glad to be able to give now this reference and to recommend Konen's monograph as perhaps the most complete treatise ever published on Doppler's effect.

It is not my intention to study theoretically the causes that can modify the optical path of the vibrations and their apparent period, nor to discuss the influence that the absolute movement of the source of light, the observer and the medium thru which the vibrations are travelling, may have on their behavior; that has been practically exhausted by Mascart, "Traité d'Optique," Kayser, Konen, Drude and many others. It is enough to say that the effect of all the causes that can change the path of the vibrations between the source of light or the absorbing medium, and the observer, must also be considered on the optical path connecting the source of the light and the absorbing element. W. Michelson's formula (*Astroph. Jour.*, **13,** 192, 1901),

$$N = n\left[1 - \frac{1}{v}\Sigma\left(l\frac{d\mu}{dt} + \mu\frac{dl}{dt}\right)\right],$$

giving the apparent period N with which the vibrations reach the observer as a function of their original period n and the change in the optical path due to the relative velocity $\frac{dl}{dt}$, between the source of light and the observer as well as to the change in refractive index $\frac{d\mu}{dt}$ in the medium, can have special application in our case.

What I intend in the following pages is to present, from the practical point of view of astrophysics, and in the most simple way, the effect that the velocity of the absorbing element with respect to the source of light can have on the absolute period of the vibrations absorbed and on the resulting changes on the structure of the discontinuous spectrum, and connect these changes with those due to velocity with respect to the observer which are considered in the ordinary application of Doppler's effect.

In order to find a general expression for all possible cases connecting *the original absolute period of the vibrations absorbed, the absolute period of the vibrations emitted by the absorbing element,* and *the relative period with which they reach the observer* (which determines their position in the spectrum), let us call S the source of light, A the absorbing element, E the observer.

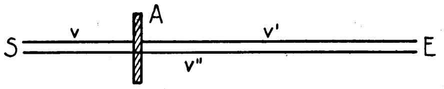

FIG. 1

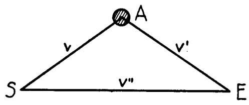

FIG. 2

Let p_x be the original absolute period of the vibrations absorbed,

p'_x be the relative period with which they arrive at the absorbing element A.

p_o be the absolute period of the vibrations emitted by the absorbing element A.

p'_o be the relative period with which vibrations of absolute period p_o coming directly from the source of light reach the observer.

p''_x be the relative period with which vibrations of absolute period p_x reach the observer.

v be the absolute value of the velocity of A with respect to S.

v' be the absolute value of the velocity of A with respect to E.

v'' be the absolute value of the velocity of S with respect to E.

Then remembering equations

$$p_x = p_o \left(1 \mp \frac{x}{c}\right)$$

$$p'_x = p_x \left(1 \pm \frac{v}{c}\right) = p_o \left(1 \mp \frac{x}{c}\right) \left(1 \pm \frac{v}{c}\right) = p_o,$$

we shall have, if c is the velocity of light,

$$p''_x = p'_x \left(1 \mp \frac{v'}{c}\right) = p_x \left(1 \pm \frac{v}{c}\right) \left(1 \mp \frac{v'}{c}\right) =$$

$$p_o \left(1 \mp \frac{x}{c}\right) \left(1 \pm \frac{v}{c}\right) \left(1 \mp \frac{v'}{c}\right) = p_o \left(1 \mp \frac{v'}{c}\right) \tag{I}$$

$$p'_o = p_o \left(1 \mp \frac{v''}{c}\right)$$

All possible values of v, v', and v'' will correspond to one of the following cases:

$$\left.\begin{array}{ll}
(1) & v = 0,\ v' = 0,\ v'' = 0, \text{ giving} \\
 & p_x = p'_x = p''_x = p_o \\
(2) & v \neq 0,\ v' \neq 0,\ v'' = 0, \text{ giving} \\
 & p_x \left(1 \pm \frac{v}{c}\right) \left(1 \mp \frac{v'}{c}\right) = p'_x \left(1 \mp \frac{v'}{c}\right) = p''_x = p_o \left(1 \mp \frac{v'}{c}\right) \\
(3) & v = 0,\ v' \neq 0,\ v'' = 0, \text{ giving} \\
 & p_x \left(1 \mp \frac{v'}{c}\right) = p'_x \left(1 \mp \frac{v'}{c}\right) = p''_x = p_o \left(1 \mp \frac{v'}{c}\right)
\end{array}\right\} \begin{array}{c}\text{and} \\ p'_o = p_o\end{array}$$

$$\left.\begin{array}{l}
(4)\quad v \neq 0,\ v' = 0,\ v'' \neq 0, \text{ giving} \\
\qquad p_x\left(1 \pm \frac{v}{c}\right) = p'_x = p''_x = p_0 \\
(5)\quad v = 0,\ v' \neq 0,\ v'' \neq 0, \text{ giving} \\
\qquad p_x\left(1 \mp \frac{v'}{c}\right) = p'_x\left(1 \mp \frac{v'}{c}\right) = p''_x = p_0\left(1 \mp \frac{v'}{c}\right) \\
(6)\quad v \neq 0,\ v' \neq 0,\ v'' \neq 0, \text{ giving} \\
\qquad p_x\left(1 \pm \frac{v}{c}\right)\left(1 \mp \frac{v'}{c}\right) = p'_x\left(1 \mp \frac{v'}{c}\right) = p''_x = p_0\left(1 \mp \frac{v'}{c}\right)
\end{array}\right\}
\begin{array}{c}\text{and}\\ p'_0 = p_0\left(1 \mp \frac{v''}{c}\right)\end{array}$$

First Case. In the first case the vibrations absorbed will be of the same period p_0 corresponding to element A, and they cannot reach the observer without their intensity being affected by the absorbing medium; the absorption lines will occupy in the spectrum their normal position corresponding to vibrations of period p_0 on the Earth.

Second Case. In the second case, which represents an absorbing medium moving between the source of light and the observer, as would be the case if vapors were projected from, or falling on, a central luminous mass, p_x cannot be equal to p_0, and therefore *emission lines due to element A can reach the observer unaffected* by the absorption of the same element if v is great enough to produce a displacement equal to the width of the line; otherwise either one of the edges of the lines can be affected according to the direction of the movement.

The main consequence of this case is that we can have *emission and absorption lines* due to the same element A; their *relative separation,* which must increase with the wave length, will be a *function* of the *velocity* with which the absorbing vapor moves with *respect to the source of light;* the emission lines from S would occupy their normal positions, v'' being zero.

Another consequence is that some lines due to an element or elements A could disappear, being absorbed by a moving element B different from the one which emits the lines. That will happen whenever there is an emission line, the corresponding period p_x of which satisfies the relation $p_x\left(1 \pm \frac{v}{c}\right) = p_0$. If the element A had vibrations of absolute periods p_0, p_1, p_2, etc., and the absorbing layer contains elements emitting vibrations of absolute period p_x, p_y, p_z, some of which satisfy the relations $p_x\left(1 \pm \frac{v}{c}\right) = p_0$, $p_y\left(1 \pm \frac{v}{c}\right) = p_1$, $p_z\left(1 \pm \frac{v}{c}\right) = p_2$, then only the emission lines for which these relations are satisfied would be absorbed or affected by the absorbing medium, the others remaining unchanged. The vibrations absorbed will reach the observer with a relative period $p''_x = p_0 \left(1 \mp \frac{v'}{c}\right)$, which fixes the position of the absorption lines in the spectrum as compared with the normal position for lines of absolute period p_0 due to element A.

In the ordinary cases indicated by Fig. 1, if $v'' = 0$ we shall have also $v' = v$ in absolute value, and then neglecting terms of the second order, we get from equation (1)

$$p_x = p''_x = p_0 \left(1 \mp \frac{v'}{c}\right),$$

showing that the absolute period p_x of the vibrations absorbed by the moving element A equals the relative period with which they reach the observer and is given by $p_0 \left(1 \mp \frac{v'}{c}\right)$, in accordance with the Doppler effect. Using the full formula as given by Michelson, we see that even for $v'' = 0$ and $v' = v$ we could have $p_x \neq p''_x$, or a difference between the original absolute period of the vibrations absorbed, and the apparent period with which they reach the observer, if the change in the optical path between the source of light and the absorbing element, were due also to terms of the form $\frac{d\mu}{dt}$, *as must happen in the case of a continuous eruptive protuberance.*

In some special cases, as in comparing the direct spectrum of the Sun with the spectrum of its light as reflected by a comet (Fig. 2), we could have $v'' = 0$ and $v' \neq v$, if the comet, surrounded by absorbing gases, were moving with different relative velocity with respect to the Sun and Earth; in such a case the constitution of the Sun's light as passing thru the comet's atmosphere would be affected by v, changing the intensity of some of the lines and possibly adding new sets of absorption lines due to elements common in the Sun's and the comet's absorbing medium.

Let us take for instance the cyanogen absorption bands *as constituted in the solar spectrum;* if they correspond to vibrations of absolute period p_0, then the relative period with which they arrive at the cyanogen of the comet will be $p_0 \left(1 \pm \frac{v}{c}\right)$, and consequently they will not be affected by the latter, which will instead absorb from the solar spectrum vibrations of period $p_x = p_0 \left(1 \mp \frac{x}{c}\right)$,* reaching the comet with a relative period $p'_x = p_o\left(1 \mp \frac{x}{c}\right)\left(1 \pm \frac{v}{c}\right) = p_o$; hence a new absorption line arises separated from the one due to the Sun's atmosphere by a distance which will be a function of the difference in periods p_o and $p_o\left(1 \pm \frac{v}{c}\right)$, no matter what might be the velocity of v', which will *shift all the lines as a whole.* We shall have in our case, if the velocity v'' (Sun-Earth) is zero, an absorption line in the direct spectrum of the Sun occupying its normal position corresponding to vibrations of period p_o, but, on the solar spectrum as coming from the comet, *two absorption lines,* the position of which will correspond to the relative periods $p_o\left(1 \pm \frac{v}{c}\right)\left(1 \mp \frac{v'}{c}\right)$ and $p_o\left(1 \mp \frac{v'}{c}\right)$, with which they reach the observer. The velocities must be taken with the positive or negative sign, according as their effect is to increase or shorten the distance between the points to which they refer.

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* When dealing with the effect of a second absorbing medium, as in the case of reflected light, P_x stands for the period of vibrations as emerging from the first absorbing layer on the Sun.

It may also be worth while to remark that even for a given value of v', the distance between the double absorption lines of the same element can vary, due to the factor $\left(1 \pm \frac{v}{c}\right)$, the effect of which is to shift the solar line, as viewed thru the comet's atmosphere, to either side of the line produced by the latter. This affords a means for distinguishing between the lines due to the reflecting body and the ones due to the original source of light.

Different sets of absorption lines due to the same element A could also appear in the spectrum of the stars, whenever on their surfaces were great masses of the element A, as eruptive protuberances, moving with radial velocities v_1, v_2, v_3, etc., each of which would absorb from the continuous spectrum, vibrations of different absolute period given by the formula $p_x = p_o\left(1 \pm \frac{x}{c}\right)$, where x stands approximately for v_1, v_2, v_3, etc. Since the light of the entire disk of the star passes thru the slit, it is evident that portions of the spectrum corresponding to vibrations with those absolute periods which have been absorbed in some regions of the stellar surface, cannot have the same intensity as the portions corresponding to vibrations which have not been absorbed in any region at all; hence if the protuberances should extend over sufficiently great regions and the difference in velocities were such as to produce a displacement greater than the *width* of the line, we would get faint and commonly narrow separated lines in triplets or in pairs due to the same element A.

The spectrum of the solar light as reflected by the planets can also be considered in this second case, more on account of their rotation than on account of the radial velocities with respect to the Sun caused by the eccentricity of their orbits. It was pointed out by H. Deslandres in a communication to the Academy of Sciences in Paris (*C. R.*, **120,** 420, 1895), that "lorsqu'un corps est éclaré par diffusion, sa lumière subit le déplacement non seulement par rapport à l'observateur, mais aussi par rapport à la source"; and in a note to this communication, H. Poincaré gave the formula $\tau\left[1+\frac{1}{c}\left(\frac{dR}{dt}+\frac{dR'}{dt}\right)\right]$ connecting the absolute period of the vibrations absorbed with the apparent period with which they reach the observer when reflected by the planet; this formula becomes identical with the one given independently in the present article, $p_o\left(1\pm\frac{v}{c}\right)\left(1\mp\frac{v'}{c}\right)$ if we put $\frac{dR}{dt}=v$ and $\frac{dR'}{dt}=v'$. As shown by J. E. Keeler (*Astroph. Jour.*, **1,** 352, 1895), the fact had been announced before, altho not with the conciseness and exactitude of Poincaré. Since then this matter has been fully discussed theoretically, and subsequently applied by different astronomers to the rotation of the planets. But here again attention has been focussed on the displacement and inclination of the lines more than on their nature and character, which depend on the relative velocity v of the observed region of the planet with respect to the Sun. An important consequence from the present point of view is that if there are elements common to the Sun's and the planet's atmospheres, their lines will coincide whenever $v = 0$, and that in all other cases *v will widen and possibly double the common lines.*

When observing *Jupiter's* east limb in opposition, the vibrations absorbed by the Sun's atmosphere with a period p_o, corresponding to the element A, would reach the observer with a relative period $p_o\left(1-\frac{12}{c}\right)\left(1-\frac{12}{c}\right)$ equal approximately to $p_o\left(1-\frac{24}{c}\right)$, while the apparent period of the vibrations absorbed by the same element of the planet, would be $p_o\left(1-\frac{12}{c}\right)$, giving a difference of relative period equal to $p_o\,\frac{12}{c}$, equivalent to a displacement of about 0.2 Å at $\lambda = 5000$ Å; with high dispersion this would be enough to separate narrow lines. Hence the most favorable conditions to detect an element common to the Sun and planet would be to observe the latter in those regions of the disk or in those places in their orbits for which v is a maximum and compare the observations with those for which v is a minimum. Let me indicate here that the further away the planet observed is, the greater will be the effect of the rotation on the broadening of the lines; the reason is that with the same width of slit, the region of the planet which reflects the light is greater, and hence v and v' can vary between wider limits.

Third Case. $v = 0$, $v' \neq 0$, $v'' = 0$. As has already been indicated, these conditions can be fulfilled in nature when observing the spectrum of the Sun's light as reflected by a planet or a satellite at the time and on the region for which v, the velocity with respect to the Sun, is zero. In such a case, both the planet's and the Sun's lines due to a common element A, will arrive at the observer with the same apparent period and consequently they will occupy the same position in the spectrum; *the absorption lines will be reinforced but not separated or resolved,* and, if compared with the lines of the direct spectrum of the Sun, all will show a shift due to the difference in periods p_0 and $p_0\left(1 \mp \frac{v'}{c}\right)$ between the direct and reflected vibrations.

Fourth Case. $v \neq 0$, $v' = 0$, $v'' \neq 0$. This is the case of an observer surrounded by an atmosphere in motion with respect to the source of light. The terrestrial absorption lines of the element A will occupy in the spectrum a normal position corresponding to vibrations of absolute period p_0, which has not been changed; but, according to the equation, p_x or the period of the vibrations absorbed from the solar spectrum, will not be the same as p_0, the period of element A in the observer's atmosphere; hence the lines cannot coincide and their separation will be a function of v or of v'', which, except for purely theoretical cases in the movement of the reflecting bodies, will be equal to v. That brings the case under the common application of Doppler's effect. In brief, the apparent period of the solar lines will be $p_0\left(1 \mp \frac{v}{c}\right)$ and that of the terrestrial lines p_0; when observing the center of the solar disk at noon v would represent the radial velocity of the Earth with respect to the Sun, which can fluctuate between ± 0.50 km. approximately. *Emission lines* of an extra-terrestrial origin and corresponding to an absolute period p_0 *can reach the observer* with the apparent period $p_0\left(1 \mp \frac{v}{c}\right)$.

Fifth Case. $v = 0, v' \neq 0, v'' \neq 0$. This is the case of a source of light surrounded by a quiet atmosphere moving as a whole with respect to the observer. As shown by the general formula applied to this hypothesis, $p_x = p_0$, and *emission lines* of absolute period p_0 due to an element A *cannot reach the observer* unaffected by its presence in the absorbing medium; they will be absorbed independently of v' and v''. If these velocities are equal, as will be the case when observing direct light, then equations $p''_x = p_0\left(1 \mp \frac{v'}{c}\right) = p_0\left(1 \mp \frac{v''}{c}\right) = p'_0$ show, in accordance with Doppler's effect, that all emission as well as absorption lines will be shifted from their normal positions by a quantity given by the difference in period p_0 and $p_0\left(1 \mp \frac{v''}{c}\right)$. If among the different periods of the vibrations emitted by the source of light, there are some given by $\frac{p_0}{1 \mp \frac{v''}{c}}$, then the relative period with which they would reach the observer, will be p_0, producing a line occupying the normal position of one due to element A, and which might be wrongly attributed to it.

Observing the Sun's light as reflected by a planet, we could have $v = 0$ and $v' \neq v'' \neq 0$. In such a case, the lines common to the Sun and the planet (or possibly to a comet) would coincide as in the third case; all the lines would be shifted from their normal positions by an amount given by the difference p_0 and $p_0\left(1 \mp \frac{v'}{c}\right)$, and if they are compared with the lines of the direct spectrum of the Sun, the displacement will be given by the difference in relative periods $p_0\left(1 \mp \frac{v''}{c}\right)$ and $p_0\left(1 \mp \frac{v'}{c}\right)$ between the direct and reflected vibrations.

Sixth Case. $v \neq 0, v' \neq 0, v'' \neq 0$ and $v \neq v' \neq v''$. These conditions can be fulfilled if a source of light is surrounded by an atmosphere, moving in a radial direction from or towards the center, and the whole having a relative velocity with respect to the observer. Here, as in the second case, *emission lines due to the element A can reach the observer*, and their separation from the absorption lines due to the same element will be given by the difference in their corresponding relative periods $p_0\left(1 \mp \frac{v''}{c}\right)$ and $p_0\left(1 \mp \frac{v'}{c}\right)$. The same expressions give also the displacement of both kind of lines as compared with the normal position due to the lines of the element A.

All the statements under second case for reflected light can be repeated here, with the only difference that the position of the lines in the direct spectrum will correspond to their relative periods $p_o\left(1 \mp \frac{v''}{c}\right)$.

Altho it is not my object to discuss the spectrograms of novae, it may be permitted to indicate some facts which according to my opinion could be explained with the preceding analysis in view:

(*a*) The presence in the spectrum of novae of emission and absorption lines due to the same element.

(*b*) The fact that these absorption lines have appeared double and even sometimes triple. (Cf. "Ueber das Spectrum der Nova Geminorum," Nr. 2, by Furnhjelm, *Publicationen des Astrophysikalischen Observatoriums zu Potsdam*, Nr. 68, 1913.)

(*c*) The fact that the absorption lines are always displaced towards the violet, at least on their first appearance, and that their relative displacements, as well as the displacements with respect to the emission lines, are proportional to the wave-length, as pointed out by various observers.

(*d*) Finally, the fact that the lines show such instability in character, seems to give weight to the assumption that we are dealing with conditions considered in the Second or the Sixth Case, to which we refer the reader.

It is true that there remains the difficulty in conceiving such high velocities as are necessary to produce shifts of over 20 A, as in the case of the absorption lines with greatest displacement; but considering: *first*, the immense proportions of the catastrophe which must take place in a nova in order to increase the intensity of its light a hundred thousand times within a few days; and *secondly*, that it is not necessary to attribute the velocity to the star as a whole (the movement of which is probably given by the displacement of the emission lines or by the narrow absorption lines H and K), but to great masses of vapor only; and *thirdly*, that even on our quiet Sun we have witnessed protuberances projected with a velocity of hundreds of kilometers per second, it does not appear so improbable that the great displacements in the absorption lines of the spectrum of novae may be due to great masses of vapor moving in radial directions from the center. The difficulty emphasized by Plaskett that "the absorbing hydrogen would have two different velocities, 1675 and 2360 kil/sec, at the same time" can be avoided by assuming that *they take place in different regions of the visible stellar surface.* If the reflection of light plays any rôle, as Seeliger and Kapteyn have suggested, then the velocities could be decreased by a considerable amount, altho not more than one-half for the most favorable conditions.

The preceding analysis may have also an application to the spectra of planetary nebulae and to the variability of bright hydrogen lines for some stars, in so far as the movement, in a radial direction, of the outer layer in a mass of radiating elements, would naturally increase the brightness of the emission lines; the reason is that in this case the relative period p'_0 of the vibrations absorbed by the cooler gases of the periphery, would not be equal to p_0, the absolute period of the vibrations emitted by the inner and hotter gases of the same elements.

Mount Wilson Observatory,
January 14, 1919.